

Anomalous Fermion Mass Generation at Three Loops

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ABSTRACT

We present a novel mechanism for generating gauge-invariant fermion masses through global anomalies at the three loop level. In a gauge theory, global anomalies are triggered by the possible existence of scalar or pseudoscalar states and heavy fermions, whose masses may not necessarily result from spontaneous symmetry breaking. The implications of this mass-generating mechanism for model building are discussed, including the possibility of creating low-scale fermion masses by quantum gravity effects.

KEYWORDS: fermion mass generation; global anomalies; higher order loop effects

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Anomalies represent a profound phenomenon in quantum physics, where symmetries that may occur in the classical action are violated by quantum-mechanical effects. The presence of anomalous terms in quantum field theory was originally noted by Steinberger [1] and subsequently by Schwinger [2]. However, it was Adler [3], and Bell and Jackiw [4] who first unravelled the remarkable properties of the so-called chiral gauge anomalies and showed their prominent role in explaining the observed sizeable decay rate of $\pi^0 \rightarrow \gamma\gamma$. Besides chiral anomalies, a number of other equally important quantum anomalies has been discovered since then, such as scaling anomalies [5, 6], gravitational anomalies [7] and the mixed gauge-gravitational anomalies [8, 9].

In this note, we present a novel radiative mechanism for generating gauge-invariant fermion masses at the three loop level, by means of perturbative chiral global anomalies. In the context of a gauge theory, the chiral global anomalies are mediated by scalar or pseudoscalar states and heavy fermions, which may possess gauge-invariant bilinear masses that do not originate from a spontaneous symmetry breaking mechanism. For illustration, we will first analyze an effective scenario for our three-loop mass-generating mechanism, in the sense that the generated fermion masses are logarithmically ultra-violet (UV) divergent and would require a UV cut-off Λ . The cut-off scale Λ could be taken to be the Planck mass $M_{\text{Pl}} = 1.9 \times 10^{19}$ GeV. After having calculated the three-loop radiative fermion mass within the framework of an effective theory with a cut-off Λ , we will then discuss a minimal UV completion of this effective scenario. Finally, we will present possible applications of the three-loop mass-generating mechanism for model-building.

To start with, we first show how the anomalous generation of an effective fermion mass arises in a simple model that can serve as a prototype for our discussion. For this purpose, we consider an anomalous-free $U(1)_V$ gauge theory with a singlet pseudoscalar state a and two Dirac fermions f and F , which both couple to a . The fermion F has a large $U(1)_V$ -invariant mass m_F , whereas the other fermion f is assumed to be massless at the tree level. The relevant Lagrangian of such a theory is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{F} (i \not{D} - m_F) F + \bar{f} i \not{D} f + \frac{1}{2} (\partial_\mu a)(\partial^\mu a) + a \left(h_F \bar{F} i \gamma_5 F + h_f \bar{f} i \gamma_5 f \right), \quad (1)$$

where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is the $U(1)_V$ field strength tensor and $D_\mu = \partial_\mu + ig_V Q_{F,f} V_\mu$ is the covariant derivative acting on the fermions F and f with hypercharges Q_F and Q_f , respectively. In addition to CP symmetry, Lagrangian (1) has one additional symmetry, in the absence of the fermion mass term $m_F \bar{F} F$. If $m_F = 0$, the Lagrangian is invariant under the discrete transformations: $a \rightarrow -a$, $F_{R(L)} \rightarrow +(-) F_{R(L)}$ and $f_{R(L)} \rightarrow +(-) f_{R(L)}$. Since we consider $m_F \neq 0$, this latter symmetry is broken softly by the dimension-3 mass operator $m_F \bar{F} F$.

Our aim is now to calculate the three-loop effective mass of the fermion f generated by virtue of the anomalous operator: $a F^{\mu\nu} \tilde{F}_{\mu\nu} \equiv a \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho}$, within a UV-cutoff effective

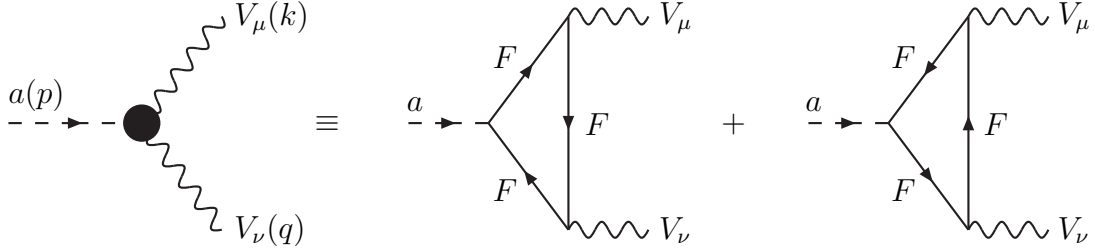


Figure 1: The five-dimensional operator $aF_{\mu\nu}\tilde{F}^{\mu\nu}$ induced by the chiral global anomaly of the heavy fermion F , with the convention $p + k + q = 0$.

theory. In other words, we will show that the first non-trivial mixing between the dimension-3 mass operators $\bar{F}F$ and $\bar{f}f$ occurs at three loops. Then, we will present a minimal UV complete extension of our simple model. Even though this is not essential for our demonstration, the pseudoscalar state a may also have a gauge-invariant bare mass and the gauge field V_μ may become massive through the usual realization of the Higgs mechanism.

In the above $U(1)_V$ gauge theory, the pseudoscalar field a couples to the gauge boson V_μ via the five-dimensional operator $aF_{\mu\nu}\tilde{F}^{\mu\nu}$. This operator is induced by the chiral global anomaly of the heavy fermion F , through the triangle graphs shown in Fig. 1. With the convention that all momenta are incoming, i.e. $p + k + q = 0$, the one-loop $a(p)$ - $V^\mu(k)$ - $V^\nu(q)$ coupling reads [1, 3, 4]:

$$i\Gamma_{\mu\nu}^{aVV}(p, k, q) = iQ_F^2 \frac{\alpha_V}{\pi} \frac{h_F}{m_F} F_P\left(\frac{p^2}{4m_F^2}\right) \varepsilon_{\mu\nu\lambda\rho} k^\lambda q^\rho, \quad (2)$$

where $\varepsilon_{\mu\nu\lambda\rho}$ is the standard anti-symmetric Levi-Civita tensor (with $\varepsilon^{0123} = +1$), $\alpha_V = g_V^2/(4\pi)$ is the fine structure constant associated with the gauge field V_μ and Q_F is the $U(1)_V$ hypercharge of the heavy fermion F . Moreover, the loop function $F_P(\tau)$ may be written down in one of the following forms [1]:

$$F_P(\tau) = \int_0^1 dx \int_0^{1-x} dy \frac{2}{1 - 4(\tau + i\epsilon)xy} = -\frac{1}{2(\tau + i\epsilon)} \int_0^1 \frac{dx}{x} \ln \left[1 - 4(\tau + i\epsilon)x(1-x) \right] \\ = \begin{cases} \frac{1}{\tau} \arcsin^2 \sqrt{\tau}; & |\tau| \leq 1, \\ -\frac{1}{4\tau} \left[\ln \left(\frac{\sqrt{\tau} + \sqrt{\tau-1}}{\sqrt{\tau} - \sqrt{\tau-1}} \right) - i\pi \right]^2; & |\tau| \geq 1, \end{cases} \quad (3)$$

with $\epsilon \rightarrow 0^+$. It is interesting to quote the small and large argument limits of the loop function $F_P(\tau)$. If $|\tau| \ll 1$, we have $F_P(\tau) = 1 + \tau/3 + \mathcal{O}(\tau^2)$, whereas in the opposite limit, $|\tau| \gg 1$, we find that $F_P(\tau) \rightarrow -\ln^2 |\tau|/(4\tau)$ is the leading term which vanishes as $\tau \rightarrow \infty$.

Our next step is to evaluate the anomalously generated radiative mass of the fermion f ,

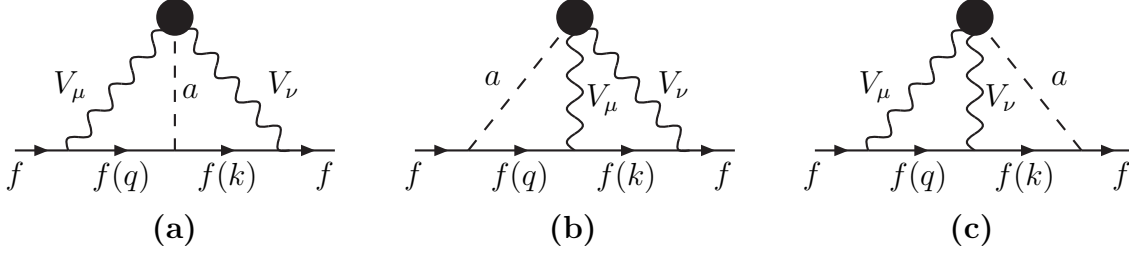


Figure 2: *Feynman graphs giving rise to anomalous fermion mass generation at three loops. Black circles denote the operator $aF_{\mu\nu}\tilde{F}^{\mu\nu}$ induced by the chiral anomaly (see also Fig. 1).*

which results from the three three-loop graphs shown in Fig. 2. First, we may convince ourselves that by naive power counting of the loop momenta, the three diagrams (a), (b) and (c) in Fig. 2 possess an overall logarithmic UV divergence, although all individual subgraphs are UV finite. Second, the three graphs together form a gauge-invariant set. Specifically, assuming a non-trivial quantization scheme of non-covariant gauge fixing, it can be shown that the gauge dependence of the V_μ -propagator in graph (a) cancels against the gauge dependence of the V_μ -propagator in graph (b), and the gauge dependence of V_ν in graph (a) against the one of V_ν in (c). Finally, there is a gauge dependence in graphs (b) and (c) that still remains, but this vanishes in their sum.

In order to reliably determine the leading contributions to the three-loop graphs in Fig. 2, we consider an effective theory with a UV cut-off energy scale $\Lambda \geq m_F$ and approximate the loop function $F_P(p^2/4m_F^2)$ given in (2) by $F_P(0) = 1$, for loop momenta less than m_F (for a related effective field-theory approach, see [10]). The result obtained for $\Lambda = m_F$ is then matched to the logarithmic UV dependence for loop momenta $\Lambda \gg m_F$. Moreover, we assume that m_F is the highest scale in the problem, i.e. $m_F \gg M_a, M_V$, which allows us to neglect all other masses. Since the three graphs depicted in Fig. 2 show the same analytical dependence on loop momenta less than $\Lambda = m_F$, the effective fermion mass m_f may be conveniently expressed as

$$m_f = i \frac{3 h_F h_f}{4\pi^2} \left(\frac{\alpha_V}{4\pi} \right)^2 \frac{Q_F^2 Q_f^2}{m_F} \gamma_5 \varepsilon_{\mu\nu\lambda\rho} \int \frac{d^4 k}{\pi^2 i} \int \frac{d^4 q}{\pi^2 i} \frac{q^\lambda k^\rho \gamma^\nu \not{k} \not{q} \gamma^\mu}{(k^2)^2 (q^2)^2 (k-q)^2}. \quad (4)$$

In order to calculate the above two-loop integral, we use the reduction formula:

$$\int \frac{d^4 q}{\pi^2 i} \frac{q_\alpha q_\beta}{(q^2)^2 (k-q)^2} = \frac{\eta_{\alpha\beta}}{4} \int \frac{d^4 q}{\pi^2 i} \frac{1}{q^2 (k-q)^2} + \frac{1}{2} \frac{k_\alpha k_\beta}{k^2}, \quad (5)$$

where $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1)$ is our convention for the Minkowski metric. Note that the second term proportional to $k_\alpha k_\beta$ in (5) vanishes when appropriately contracted with the anti-symmetric expression $\varepsilon_{\mu\nu\lambda\rho} k^\lambda$ in (4). Hence, the reduction formula (5) can be applied twice, first for the loop momentum q and then for k . In this way, the effective fermion mass expression (4)

simplifies to:

$$m_f = \frac{3 h_F h_f}{2\pi^2} \left(\frac{\alpha_V}{4\pi} \right)^2 \frac{Q_F^2 Q_f^2}{m_F} \int \frac{d^4 k}{\pi^2 i} \int \frac{d^4 q}{\pi^2 i} \frac{1}{k^2 q^2 (k-q)^2}, \quad (6)$$

where we also used the fact that $\gamma_5 = \frac{i}{4!} \varepsilon_{\mu\nu\lambda\rho} \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\rho$. By means of a standard Wick rotation, we can analytically calculate the two-loop integral occurring in (6). Using spherical coordinates to parameterize the volume element of a four-dimensional Euclidean sphere of radius Λ , we obtain

$$I_\Lambda = \int \frac{d^4 k}{\pi^2 i} \int \frac{d^4 q}{\pi^2 i} \frac{1}{k^2 q^2 (k-q)^2} = -(4 \ln 2) \Lambda^2. \quad (7)$$

The integral I_Λ is infra-red safe and scales quadratically with Λ , as expected by naive power counting. Substituting (7) into (6) and matching the resulting expression at $\Lambda = m_F$, we arrive at a simple formula for the effective fermion mass of f ,

$$m_f = -6 \frac{h_F h_f}{\pi^2} \left(\frac{\alpha_V}{4\pi} \right)^2 Q_F^2 Q_f^2 m_F \ln \left(\frac{\Lambda^2 + m_F^2}{m_F^2} \right). \quad (8)$$

In obtaining (8), we have also included the logarithmic factor $\ln[(\Lambda^2 + m_F^2)/m_F^2]$ to properly describe the UV dependence of m_f on the UV cut-off Λ . Assuming a UV cut-off $\Lambda = M_{\text{Pl}}$, an ultra-heavy fermion F , with mass $m_F = 10^{13}$ TeV close to the GUT scale, an SO(10)-type gauge coupling g_V , with $\alpha_V = 1/30$ and perturbative Yukawa couplings $h_{F,f} = 10^{-2}$, one gets a three-loop effective fermion mass $m_f \approx 2 \times 10^{-9} m_F = 2 \times 10^4$ TeV, for $Q_F = Q_f = 1$. If the pseudoscalar state a has a mass $M_a \gtrsim m_F$, then there will be an extra suppression factor of m_F^2/M_a^2 in (8), viz.

$$m_f \sim \frac{h_F h_f}{\pi^2} \left(\frac{\alpha_V}{4\pi} \right)^2 Q_F^2 Q_f^2 \frac{m_F^3}{M_a^2} \ln \left(\frac{\Lambda^2}{M_a^2} \right). \quad (9)$$

Thus, a mild hierarchy of about two orders of magnitude, e.g. $m_F/M_a \sim 0.01$, can lead to singlet fermion masses m_f at the TeV scale.

It is important to explore whether the above effective $U(1)_V$ scenario can be made UV complete. A minimal extension is to consider two pseudoscalars a_f and a_F , which couple to fermions f and F , respectively. Specifically, the Yukawa sector of the model is extended, so as to assume the form

$$\mathcal{L}_Y = h_f i a_f (\bar{f}_L f_R - \bar{f}_R f_L) + h_F i a_F (\bar{F}_L F_R - \bar{F}_R F_L) - m_F (\bar{F}_L F_R + \bar{F}_R F_L). \quad (10)$$

The Yukawa Lagrangian \mathcal{L}_Y possesses two exact symmetries: (i) CP invariance and (ii) an f -chiral discrete symmetry, where $a_f \rightarrow -a_f$ and $f_L \rightarrow -f_L$, whilst the fields a_F , f_R and $F_{L,R}$ do not transform. It is easy to promote these two symmetries to the gauge kinetic part

of the Lagrangian. Instead, we explicitly break the f -chiral discrete symmetry in the scalar potential V by a mixing-mass operator $a_f a_F$ of dimension-2, i.e.

$$V(a_f, a_F) = \frac{1}{2} M_a^2 (a_f^2 + a_F^2) + \delta M_a^2 a_f a_F. \quad (11)$$

For simplicity, we have taken here the bilinear mass parameters for a_f^2 and a_F^2 equal to M_a^2 , with $M_a^2 > \delta M_a^2$. Consequently, the higher dimension-3 fermion-mass operator $m_f \bar{f} f$ that breaks the second symmetry (ii) will be generated at the three loop level and it will be UV finite, as we explain below.

The only channel of communication between the F - and f - sectors is through the mixing mass term $\delta M_a^2 a_f a_F$ in the scalar potential (11). A UV finite three-loop mass for the fermion f will be generated by the Feynman graphs shown in Fig. 2, where the a -loop exchange line should be replaced with the pseudoscalar transition $a_F \rightarrow a_f$. Because of the latter modification, the overall degree of divergence of the three-loop graphs reduces to -2 , leading to a UV finite result. Assuming that $m_F \gg M_a$, we may estimate the three-loop fermion mass of f to be

$$m_f \sim \frac{h_F h_f}{\pi^2} \left(\frac{\alpha_V}{4\pi} \right)^2 Q_F^2 Q_f^2 \frac{\delta M_a^2}{M_a^2} m_F. \quad (12)$$

Evidently, if we choose as before the mass of the ultra-heavy fermion to be $m_F = 10^{13}$ TeV, but $h_{F,f} = 1$, we then obtain fermion mass $m_f \sim 1$ TeV, for $\delta M_a^2/M_a^2 = 10^{-8}$.

It is interesting to discuss the key differences of a U(1) model with a CP-even singlet scalar σ , instead of a CP-odd boson a . Like the pseudoscalar a , a CP-even singlet scalar σ may also give rise to anomalous fermion mass generation, through the operator $\sigma F_{\mu\nu} F^{\mu\nu}$ that breaks anomalously the scaling symmetry [5, 6]. In this case, however, the presence of the explicit scale-violating mass term $m_F \bar{F} F$ of dimension-3 requires that other explicit scale-violating operators up to the same dimension be added to Lagrangian (1), such as the tadpole σT_σ , the bilinear mass $M_\sigma^2 \sigma^2$ and the trilinear term $\mu_\sigma \sigma^3$, in order for the theory to remain renormalizable. One may consider tuning the tadpole parameter T_σ , such that the singlet field σ has a zero vacuum expectation value (VEV) to all orders in perturbation theory. Such a renormalization condition would prevent f from acquiring a mass, through the usual Higgs mechanism. However, there will still be a two-loop contribution [11, 12] induced by the trilinear term $\mu_\sigma \sigma^3$ that gives rise to a UV-divergent local mass for the fermion f . Moreover, the trilinear term $\mu_\sigma \sigma^3$ requires renormalization beyond the tree-level, as it receives UV-divergent corrections from F -fermion one-loop graphs. Notice that in a CP-invariant theory containing CP-odd scalars $a_{f,F}$ only, model parameters analogous to the tadpole T_σ and the trilinear coupling μ_σ are absent to all orders in perturbation theory [13]. For this reason, the observed CP violation in the quark (and possibly lepton) sector can only be explained within models that realize spontaneous breaking of CP symmetry, e.g. in two Higgs doublet models [14] (for

a recent review, see [15]). An additional requirement is that the ground states $a_{f,F}$ have zero VEVs, after spontaneous symmetry breaking.

The three-loop mass-generating mechanism may have a potentially interesting application in a $U(1)_{B-L}$ extension of the Standard Model (SM). To be specific, we extend the particle content of the SM by the addition of 5 singlet Weyl neutrinos per lepton family l : $n_{1L,R}$, $n_{2L,R}$ and the right-handed neutrino ν_R . All the singlet neutrinos carry lepton number. In the weak basis $[(\nu_L)^C, \nu_R, (n_{1L})^C, n_{1R}, (n_{2L})^C, n_{2R}]$, the neutrino mass matrix M^ν may be cast into the form:

$$M^\nu = \begin{pmatrix} 0 & m_D & 0 & 0 & 0 & m'_D \\ m_D & 0 & M^{(1)} & 0 & M^{(2)} & 0 \\ 0 & M^{(1)} & 0 & M_1 & 0 & M^{(3)} \\ 0 & 0 & M_1 & 0 & M_2 & 0 \\ 0 & M^{(2)} & 0 & M_2 & 0 & M^{(4)} \\ m'_D & 0 & M^{(3)} & 0 & M^{(4)} & M_{B-L} \end{pmatrix}. \quad (13)$$

Here, $M_{1,2}$ are GUT-scale mass parameters that occur at the tree level, whilst the parameters $M^{(1,2,3,4)}$ are generated via the three-loop mechanism discussed above and can be as low as TeV, according to the estimate given in (12). In addition, we considered the possibility of spontaneous breaking of $U(1)_{B-L}$ by the right-handed neutrinos n_2 , where the scale M_{B-L} could be close to GUT scale as well. The structure of the mass matrix M^ν may enforced by demanding that the fermion fields ν_L, ν_R, n_{2R} and the pseudoscalar a_f are odd under a discrete transformation, whereas all other neutrino states $n_{1L,R}, n_{2L}$ and the pseudoscalar state a_F are even. Integrating out the heavy states $n_{1,2}$ and assuming that $m'_D \ll m_D$, we obtain the low-energy effective neutrino mass matrix

$$M_{\text{eff}}^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M^{(1)} \\ 0 & M^{(1)} & \mu_{B-L} \end{pmatrix}, \quad (14)$$

where $\mu_{B-L} \sim (M^{(3)})^2/M_{B-L}$ is an effective $(B-L)$ -violating mass term. The resulting low-energy theory strongly resembles the inverse seesaw model with pseudo-Dirac TeV-mass heavy neutrinos [16, 17], where all the isosinglet masses are induced radiatively. Likewise, a TeV-mass pattern could also be obtained for theories with vector quarks. The discussion goes along similar lines and we do not repeat it here.

It is now interesting to observe that even in the absence of a $U(1)_V$ gauge group, quantum gravity interactions may be sufficient to mediate the chirality-violating effects of the ultra-heavy fermion F to the low-energy sector of the theory. Specifically, one may envisage a scenario where the role of the gauge bosons V_μ is played by the gravitons $h_{\mu\nu}$ within a linearized framework of quantum gravity. Then, integrating out the heavy fermion F in graphs analogous to Fig. 1, an effective coupling of the pseudoscalar field a to gravity will be generated, through the operator

$a \varepsilon^{\mu\nu\lambda\rho} R_{\alpha\beta\mu\nu} R^{\alpha\beta}_{\lambda\rho}$, where $R_{\alpha\beta\mu\nu}$ is the Riemann tensor. We note that the suggested mechanism is perturbative pertaining to a matter pseudoscalar a and it is not necessarily related to the form-valued Kalb-Ramond axions [18, 19]. Assuming an effective cut-off energy scale $\Lambda = m_F$, we may perform a naive dimensional analysis of the respective three-loop graphs in Fig. 2 to estimate m_f as

$$m_f \sim h_f h_F \frac{m_F^9}{m_{\text{Pl}}^8} \approx 10^{-3} \times h_f h_F \left(\frac{m_F}{10^{16} \text{ GeV}} \right)^9 \text{ GeV}, \quad (15)$$

where $m_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} \approx 2.4 \times 10^{18} \text{ GeV}$ is the reduced Planck mass. The estimate (15) is obtained by observing that the strength of each gravitational coupling in the loop is proportional to $1/m_{\text{Pl}}^2$. If $h_F = h_f \sim 1$ and $m_F = 4 \times 10^{16} \text{ GeV}$, we find an effective radiative mass $m_f \sim 250 \text{ GeV}$, induced by ordinary quantum gravity effects. This result is very sensitive to the mass m_F and the actual UV completion of quantum gravity. Nevertheless, it is amusing to note that for $m_F = 10^{16} \text{ GeV}$, one may account for isosinglet masses m_f at the keV range. In particular, it was argued [20] that keV-mass sterile neutrinos may successfully play the role of warm dark matter.

It would be worth investigating whether a supersymmetric realization of our anomalous three-loop mechanism exists. Let us therefore consider an anomalous-free $U(1)_V$ model with one chiral singlet superfield \hat{S} and two pairs of oppositely charged chiral superfields $\hat{F}_{L,R}$ and $\hat{f}_{L,R}$. The pertinent superpotential of this model is given by

$$W = h_F \hat{S} (\hat{F}_L \hat{F}_R - m_F^2) + M_S \hat{S}^2 + h_f \hat{S} \hat{f}_L \hat{f}_R. \quad (16)$$

In the absence of the term m_F^2 , the superpotential possesses an exact R symmetry, with R charges: $R(\hat{S}) = 1$, $R(\hat{F}_{L,R}) = R(\hat{f}_{L,R}) = 1/2$ and $R(W) = 2$. The superpotential tadpole parameter m_F^2 breaks softly the R symmetry and may effectively be generated by supergravity effects (see, e.g. [21] and references therein). For this reason, we assume that both m_F and M_S are large GUT-scale parameters. On the other hand, according to the non-renormalization theorem in supersymmetry, a superpotential mass term $m_f \hat{f}_L \hat{f}_R$ cannot be generated by *renormalizable* supersymmetric interactions to all orders in perturbation. Also, the existence of such a term would break the R symmetry softly by a dimension-3 operator.

In the supersymmetric limit, one possible solution to the ground state of the above supersymmetric model consists of having the singlet superfield \hat{S} with vanishing VEV², whilst the $U(1)_V$ -charged chiral superfields $\hat{F}_{L,R}$ develop a non-zero VEV, i.e. $\langle \hat{F}_L \rangle = \langle \hat{F}_R \rangle = m_F$, thus breaking $U(1)_V$ spontaneously. The other charged chiral superfields $\hat{f}_{L,R}$ have vanishing VEVs and so their fermion components remain massless in the supersymmetric limit. Instead,

²Soft supersymmetry breaking effects may generate a non-zero VEV $\langle S \rangle \sim (A_F - \xi_S)/h_F$, which can be naturally small of the electroweak order, or even tuned to zero, where A_F and ξ_S are the soft supersymmetry breaking parameters associated with the trilinear coupling $\hat{S}\hat{F}_L\hat{F}_R$ and the tadpole \hat{S} .

the corresponding fermion fields of $\widehat{F}_{L,R}$ combine with the fermionic component of \widehat{S} and the gauginos of $U(1)_V$ to form heavy Dirac fermions of masses of order m_F and M_S . Integrating out these heavy fermions, we obtain the 5-dimensional operator $\widehat{S}\widehat{W}^\alpha\widehat{W}_\alpha$, where \widehat{W}_α is the chiral superfield strength spinor related to the $U(1)_V$ gauge group. Even though this operator is analogous to the one generated in Fig. 1, the non-renormalization theorem in supersymmetry prohibits the generation of a three-loop mass by loop graphs similar to Fig. 2 and their supersymmetric counterparts. However, non-renormalizable supergravity interactions might be sufficient to generate a sizeable effective radiative mass for m_f , of the electroweak order, according to the estimate (15), when the mass parameters m_F and M_S are taken to be sufficiently large, e.g. in the vicinity of the gauge coupling unification scale $M_X \sim 2 \times 10^{16}$ GeV. An extensive calculation lies beyond the scope of this note and may given elsewhere.

In summary, we have presented a novel radiative mechanism for generating gauge-invariant fermion masses through global anomalies at the three loop level. A minimal, UV-complete realization of this mechanism requires the presence of at least two singlet pseudoscalar states which could couple to gauge-invariant fermion bilinears. We have assumed that one family F of fermion bilinears breaks chirality at a very high scale, e.g. close to the GUT scale. In a gauge theory, this chirality violation can be mediated to another family f of fermions by the three-loop graphs shown in Fig. 2, producing masses that could be of the electroweak order, according to our estimate in (12). This mechanism can be used to naturally explain the possible existence of heavy neutrinos at the electroweak or even lower scale. In addition, we have discussed its potential implications for model building, including the possibility of creating low-scale fermion masses by quantum gravity effects in non-supersymmetric and supersymmetric theories. Further studies are needed in this direction, in order to be able to assess the full range of applicability of the proposed mechanism.

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References

- [1] J. Steinberger, “On the Use of Subtraction Fields and the Lifetimes of Some Types of Meson Decay,” *Phys. Rev.* **76** (1949) 1180.
- [2] J. S. Schwinger, “The Theory of Quantized Fields I,” *Phys. Rev.* **82** (1951) 914.
- [3] S. L. Adler, “Axial Vector Vertex in Spinor Electrodynamics,” *Phys. Rev.* **177** (1969) 2426.
- [4] J. S. Bell and R. Jackiw, “A PCAC puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the Sigma Model,” *Nuovo Cim. A* **60** (1969) 47.
- [5] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, “A Phenomenological Profile of the Higgs Boson,” *Nucl. Phys. B* **106** (1976) 292.
- [6] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, “Remarks on Higgs Boson Interactions with Nucleons,” *Phys. Lett. B* **78** (1978) 443.
- [7] L. Alvarez-Gaume and E. Witten, “Gravitational Anomalies,” *Nucl. Phys. B* **234** (1984) 269.
- [8] W. A. Bardeen and B. Zumino, “Consistent and Covariant Anomalies in Gauge and Gravitational Theories,” *Nucl. Phys. B* **244** (1984) 421.
- [9] L. Alvarez-Gaume and P. H. Ginsparg, “The Structure of Gauge and Gravitational Anomalies,” *Annals Phys.* **161** (1985) 423 [Erratum-ibid. **171** (1986) 233].
- [10] A. Dedes and K. Suxho, “Heavy Fermion Non-Decoupling Effects in Triple Gauge Boson Vertices,” *arXiv:1202.4940 [hep-ph]*.
- [11] J. F. Nieves, “Baryon And Lepton Number Nonconserving Processes And Intermediate Mass Scales,” *Nucl. Phys. B* **189** (1981) 182.
- [12] A. Zee, “Quantum Numbers of Majorana Neutrino Masses,” *Nucl. Phys. B* **264** (1986) 99.
- [13] A. Pilaftsis, “CP-Odd Tadpole Renormalization of Higgs Scalar–Pseudoscalar Mixing,” *Phys. Rev. D* **58** (1998) 096010.
- [14] T. D. Lee, “A Theory of Spontaneous T Violation,” *Phys. Rev. D* **8** (1973) 1226.
- [15] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, “Theory and Phenomenology of Two Higgs Doublet Models,” *Phys. Rept.* **516** (2012) 1.
- [16] R. N. Mohapatra and J. W. F. Valle, “Neutrino Mass and Baryon Number Nonconservation in Superstring Models,” *Phys. Rev. D* **34** (1986) 1642.

- [17] S. Nandi and U. Sarkar, “A Solution to the Neutrino Mass Problem in Superstring E6 Theory,” *Phys. Rev. Lett.* **56** (1986) 564.
- [18] M. Kalb and P. Ramond, “Classical Direct Interstring Action,” *Phys. Rev. D* **9** (1974) 2273.
- [19] M. J. Bowick, S. B. Giddings, J. A. Harvey, G. T. Horowitz and A. Strominger, “Axionic Black Holes and a Bohm-Aharonov Effect for Strings,” *Phys. Rev. Lett.* **61** (1988) 2823.
- [20] T. Asaka, M. Shaposhnikov and A. Kusenko, “Opening a New Window for Warm Dark Matter,” *Phys. Lett. B* **638** (2006) 401.
- [21] J. Bagger, E. Poppitz and L. Randall, “Destabilizing Divergences in Supergravity Theories at Two Loops,” *Nucl. Phys. B* **455** (1995) 59.